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THE BJORKEN SYSTEM OF EQUATIONS AND NUCLEON SPIN STRUCTURE-FUNCTION DATA *

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ABSTRACT

The status of the Bjorken sum rule is examined in the light of recent data on the spin structure functions of the deuteron and proton obtained by the SMC group, the neutron by the E142 group and the proton by the E143 group. Combining the new data with that already obtained for the proton by the EMC group and SLAC/YALE collaborations, we show that the Bjorken system of equations is violated at the $2{-}3\sigma$ level. We also discuss in detail the role of possible higher-twist contributions and higher-order PQCD corrections.

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Polarisation effects in general can provide valuable insight into the dynamics of hadronic interactions and are extremely sensitive to the bound-state structure, so elusive to theoretical approach.[1] In particular, the Bjorken sum rule (BSR)[2] is a measurable quantity that can be used to test theoretical predictions. The experimental precision now attainable is at the ten-percent level while, on the theoretical side, all relevant PQCD calculations have been carried out to two-loop order[3] (i.e., one-percent level) and for the BSR itself to three loops.[4] Thus, one can consider such comparisons as serious, indeed obligatory, tests of the applicability of PQCD to such processes.

In the quark-parton model (QPM) the structure function $g_1(x,Q^2)$[5] is simply related to polarised quark distributions, analogous to those for $F_1(x,Q^2)$:

$$g_1(x,Q^2) = \tfrac{1}{2}\sum_f e_f^2\,\Delta q_f(x,Q^2) \qquad \text{and} \qquad F_1(x,Q^2) = \tfrac{1}{2}\sum_f e_f^2\,q_f(x,Q^2). \tag{1}$$

The quark densities are defined in the following manner:

$$\Delta q_f(x,Q^2) = q_f^+(x,Q^2) - q_f^-(x,Q^2) \qquad \text{and} \qquad q_f(x,Q^2) = q_f^+(x,Q^2) + q_f^-(x,Q^2). \tag{2}$$

where $q_f^\pm(x,Q^2)$ are the densities of quarks of flavour f and positive or negative helicity with respect to the parent hadron.

Experimentally one measures an asymmetry, the polarised structure function is then extracted via

$$g_1(x,Q^2) = \frac{A_1(x,Q^2)\,F_2(x,Q^2)}{2x\,(1+R(x,Q^2))}, \tag{3}$$

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where $R_1(x, Q^2)$ is the ratio of longitudinal to transverse unpolarised structure functions and $A_1(x, Q^2)$ is the measured asymmetry.

The Bjorken sum rule[2] with PQCD corrections reads

$$\Gamma_1^{p-n} = \int_0^1 dx\, g_1^{p-n}(x, Q^2) = \tfrac{1}{6}\, g_A \left[1 - \alpha_s/\pi - c_2(\alpha_s/\pi)^2 - \ldots\right], \tag{4}$$

the coefficients being known to third order.[4]

The full SU(3) algebra of the baryon octet admits three independent quantities, which may be expressed in terms of the SU(3) axial-vector couplings:

$$\begin{aligned}
\langle\, p^\uparrow \mid \bar{u}\gamma_3\gamma_5 u - \bar{d}\gamma_3\gamma_5 d \mid p^\uparrow\,\rangle &= \langle\, p^\uparrow \mid \mathcal{A}_3 \mid p^\uparrow\,\rangle = g_A, \\
\langle\, p^\uparrow \mid \bar{u}\gamma_3\gamma_5 u + \bar{d}\gamma_3\gamma_5 d - 2\bar{s}\gamma_3\gamma_5 s \mid p^\uparrow\,\rangle &= \langle\, p^\uparrow \mid \mathcal{A}_8 \mid p^\uparrow\,\rangle = \tilde{g}_A, \\
\langle\, p^\uparrow \mid \bar{u}\gamma_3\gamma_5 u + \bar{d}\gamma_3\gamma_5 d + \bar{s}\gamma_3\gamma_5 s \mid p^\uparrow\,\rangle &= \langle\, p^\uparrow \mid \mathcal{A}_0 \mid p^\uparrow\,\rangle = g_0.
\end{aligned} \tag{5}$$

The right-hand sides of the first two equations correspond to measured constants (g_A=1.2573±0.0028[6] and $\tilde{g}_A$=0.629±0.039[7]), but the third (g_0), corresponding to the flavour-singlet axial-vector current, is unknown. Thus a direct prediction for, say, just the proton integral is not possible. A further combination of the u, d and s axial-current matrix elements is accessible in ν-p elastic scattering[8] and thus would allow an exact prediction for single nucleon targets. Unfortunately, the precision of such measurements is still very poor.

However, good arguments can be made for setting the strange-quark matrix element equal to zero[9]: there are few strange quarks in the proton and they are concentrated below $x_B \simeq 0.1$, where all correlations are expected to die out. Thus, the last two matrix elements of eqs. (5) might be expected to be equal, leaving only two independent quantities and allowing predictions for the proton and neutron separately:

$$\Gamma_1^{p(n)} = (-)\tfrac{1}{12} g_A + \tfrac{5}{36}\tilde{g}_A + \tfrac{1}{3}\langle\, p^\uparrow \mid \bar{s}\gamma_3\gamma_5 s \mid p^\uparrow\,\rangle, \tag{6}$$

where the last term is then assumed negligible. For clarity, the PQCD corrections have been suppressed. Conversely, these equations may be used to extract the value of either the strange-quark or singlet axial-vector matrix element, given the value of Γ_1. There is no space here to discuss the problem of the strange-quark spin; the interested reader is referred to,[10,11] where a bound on the non-diffractive component and thus on the strange-quark polarisation was derived. The result of this analysis is the following bound: $|\int \Delta s| \leq 0.02$.

We now compare the results obtained by the three experiments with theoretical predictions based on the above. In performing the calculations we have used the very precise value of $\Lambda^{(4)}_{\rm QCD}$ recently extracted in a three-loop analysis of scaling violations in deep-inelastic scattering (DIS),[12] which is thus most suitable for our purposes. This analysis also allows an examination of the improvement obtained on increasing the order of the perturbation theory analysis. Let us take the opportunity to stress that for any analysis to be consistent, all quantities involved must be evaluated at the same

loop order and that, in particular, it is meaningless to insert a two-loop α_s into a three-loop expression.

$$
\begin{array}{llcl}
\mathrm{EMC}^{13} & \Gamma_1^p(11\,\mathrm{GeV}^2) & = & 0.126 \pm 0.010 \pm 0.015 \\
\mathrm{SMC}^{14} & \Gamma_1^p(10\,\mathrm{GeV}^2) & = & 0.136 \pm 0.011 \pm 0.011 \\
\mathrm{E143}^{15} & \Gamma_1^p(3\,\mathrm{GeV}^2) & = & 0.133 \pm 0.004 \pm 0.012 \\
\mathrm{SMC}^{16} & \Gamma_1^d(5\,\mathrm{GeV}^2) & = & 0.023 \pm 0.020 \pm 0.015 \\
\mathrm{E142}^{17} & \Gamma_1^n(2\,\mathrm{GeV}^2) & = & -0.022 \pm 0.006 \pm 0.009
\end{array}
\tag{7}
$$

$$
\begin{array}{llcl}
 & \Gamma_1^p(11\,\mathrm{GeV}^2) & = & 0.182 \pm 0.006 + \frac{1}{3}\int\Delta s \\
\text{Ellis-} & \Gamma_1^p(10\,\mathrm{GeV}^2) & = & 0.182 \pm 0.006 + \frac{1}{3}\int\Delta s \\
\text{Jaffe} & \Gamma_1^p(3\,\mathrm{GeV}^2) & = & 0.179 \pm 0.006 + \frac{1}{3}\int\Delta s \\
 & \Gamma_1^d(5\,\mathrm{GeV}^2) & = & 0.085 \pm 0.006 + \frac{1}{3}\int\Delta s \\
 & \Gamma_1^n(2\,\mathrm{GeV}^2) & = & -0.010 \pm 0.006 + \frac{1}{3}\int\Delta s
\end{array}
\tag{8}
$$

The short-fall in the proton measurements with respect to the Ellis-Jaffe prediction (taking $\int\Delta s{=}0$) is immediately obvious. This observation led to the coining of the phrase *Spin Crisis*. A similar (though less striking) observation may be made for the SMC deuteron integral. In contrast, the neutron sum rule appears well satisfied by the E142 data. In terms of the strange-quark contribution, both the EMC and SMC measurements imply $\int\Delta s{\simeq}-0.15$ while that of E142 leads to $\int\Delta s{\simeq}-0.04$.

A measure of the discrepancy between the data and theory may be obtained by extracting the singlet axial-vector matrix element: the results are

$$
\begin{array}{rll}
\Delta q & = 0.14 \pm 0.16 & \text{EMC proton} \\
 & = 0.21 \pm 0.14 & \text{SMC proton} \\
 & = 0.21 \pm 0.12 & \text{E143 proton} \\
 & = 0.06 \pm 0.22 & \text{SMC deuteron} \\
 & = 0.20 \pm 0.08 & \text{global proton} \\
 & = 0.51 \pm 0.10 & \text{E142 neutron,}
\end{array}
\tag{9}
$$

where Δq is the sum of quark polarisations as in eqs. 5. Alternatively, one can fit for the strange-quark spin contribution.[18] Taking the SLAC proton and neutron data and performing completely consistent fits at one- two- and three-loop order we obtain respectively $\chi^2{=}3.7$, 3.8 and 3.2 for one degree of freedom. Using the Particle Data Group[6] preferred value of $\Lambda^{(4)}_{\mathrm{QCD}}{=}260^{+56}_{-46}$ in a two-loop fit (for consistency with the extraction of Λ), the situation is marginally improved to give $\chi^2{=}2.8$.

Given the low Q^2 of the SLAC data, one should naturally worry about the possibility of higher-twist "contamination". There are two approaches to this problem: either one attempts to estimate theoretically the size of such effects (e.g., using a bag model[19] or QCD sum rules[20]) or one deduces limits from the well-documented higher-twist behaviour of the unpolarised data.[21] In either case it turns out that the magnitude of

higher-twist contributions to DIS is far too small to have any real impact, even on the SLAC neutron data (by a strange quirk, the higher-twist contribution to g_1^n is typically much smaller even than that in the case of g_1^p).

It is interesting to ask what occurs if the normalization condition on the Wilson coefficients is relaxed, i.e., if one ignores PQCD and uses current algebra only to fix ratios of matrix elements.[21] In this case, using our strange-quark bound to effectively set Δs=0, any one data set may be used to fix the overall normalization. The EMC proton data, for example, then lead to the following "prediction" for the neutron: $0.002 \leq \Gamma_1^n \leq -0.026$, in rather good agreement with the SLAC data. Alternatively, the quark spins may be deduced from the proton and neutron data: one arrives at the following relation:

$$\Gamma_1^n = -\tfrac{1}{11}\Gamma_1^p + \tfrac{2}{3}\Delta s, \tag{10}$$

which leads to $\Delta s = -0.03 \pm 0.03$, again perfectly compatible with our bound.

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